Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Mass Transfer 50 (2007) 4559–4567
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Controllability of cross-flow two-phase heat exchangers

Gerardo Diaz *

School of Engineering, University of California, Merced, P.O. Box 2039, Merced, CA 95344, USA

Received 11 July 2006; received in revised form 17 March 2007
Available online 22 May 2007

Abstract

The analysis of controllability of a heat exchanger operating under two-phase conditions is performed. An evaporator operating in a vapor compression system is chosen for the analysis. A moving-boundary model is used to simulate the dynamic behavior of the evaporator. First, the controllability of the linearized model is verified. Then, it is shown that the nonlinear model can be classified as a control-affine system with drift. Short-term local controllability of the nonlinear model is shown using Lie algebras. A linear quadratic (LQ) controller is developed for the linearized system and numerical simulations are provided that show a strong coupling between the variables of the model.

Keywords: Controllability; Two-phase; Heat exchangers

1. Introduction

Heat exchangers operating under two-phase flow conditions are commonly used in a wide range of industries, including, among others, power generation, food processing, chemical plants, refrigeration, desalination, and transportation. A familiar example of the utilization of two-phase flow heat exchangers is a vapor compression cycle. These cycles are commonly used for air conditioning and refrigeration purposes. They are composed of a condenser, compressor, expansion device, and evaporator. Transcritical-cycle vapor compression systems use a gas cooler instead of a condenser and they have one extra heat exchanger called suction line heat exchanger that is used to improve the efficiency of the cycle. Fig. 1 shows a schematic of both systems. These systems are usually designed to meet certain performance and cost specifications under steady-state conditions. Once the components of the system have been assembled together, a control logic is developed to take the system to different operating conditions and also to reject external disturbances. In general, the control logic is obtained considering the dynamic response of the assembled system subject to external stimuli. Before developing and tuning the control logic for the assembled system, it is clearly desirable to know if the individual components can be controlled, i.e. if we can take any of the components from any single state to any given state during a prescribed time interval. This can be done by analyzing the controllability of a component. Since the compressor and the expansion device have much faster reactions than the heat exchangers [1], they can be modeled using algebraic equations. Controllability of single-phase cross-flow heat exchangers has been analyzed in the past [2], this paper addresses the controllability of cross-flow heat exchangers operating under two-phase flow conditions. Since the high-pressure side of the vapor compression cycle can have a condenser or a gas cooler, for which there is no two-phase flow, we will consider only the evaporator in our analysis since this component always operates in two-phase.

Air conditioning systems are used to remove latent and sensible heat from an air-stream. The system has to be operated so that the external surface of the evaporator is prevented from freezing. The operating conditions of the air conditioning system can be manipulated by different

* Tel.: +1 209 228 7858; fax: +1 209 228 4047.
E-mail address: gdiaz@ucmerced.edu

0017-9310/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2007.03.024

Nomenclature

A	area
$\mathbf{A}$	matrix operator in linearized system
$\widehat{A}$	matrix operator in enhanced system
$\mathbf{B}$	matrix operator for manipulated variable in linearized system
$\widehat{B}$	matrix operator of manipulated variable in enhanced system
C_p	specific heat
D	diameter
$\mathscr{F}$	function of $\mathbf{x}$ and $\mathbf{u}$
$\widehat{G}$	matrix operator of reference variable in enhanced system
P	pressure
$\mathbf{Q_c}$	controllability matrix
S	solution of Riccati equation
T	temperature
$\mathbf{W}$	matrix of vector fields
$\mathbf{Z}$	non-singular matrix
$\mathbf{f}$	drift vector
$\mathbf{g}$	smooth vector field
h	enthalpy
l	length of two or single phase
$\dot{m}$	mass flow rate
n	number of states

Greek symbols

α	heat transfer coefficient
$\bar{\gamma}$	mean void fraction
ρ	density

Subscripts and superscripts

1	two-phase section
2	single-phase section
a	ambient
g	vapor
i	inlet
int	interface
o	outlet
r	refrigerant
w	wall

mechanisms. Some air conditioning systems have a capillary tube to create the pressure drop between the high and low pressure side. There is no possibility to manipulate this device, so the compressor is run at constant speed and turned on and off to maintain a certain desired temperature. Other systems have a thermal expansion valve that can be opened or closed to vary the amount of superheat at the outlet of the evaporator. Transcritical systems use an electronic expansion valve. Also, some systems allow the fan speed near the evaporator to be varied manually or automatically. Since the compressor speed is either fixed, as in residential systems, or it follows the RPMs of the engine, as in automotive systems, it is not considered a manipulated variable. The thermal expansion valve is used to control the amount of superheat at the exit of the evaporator, and the change in the air flow rate varies the heat transfer rate across the evaporator.

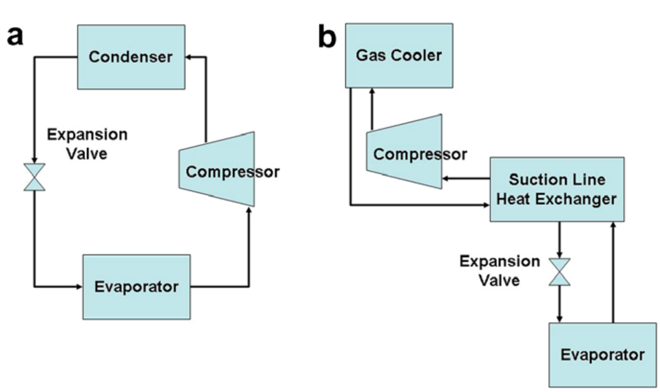

Fig. 1. Typical vapor compression systems: (a) standard and (b) transcritical.

To evaluate the controllability of the evaporator, dynamic models that simulate the behavior of this heat exchanger have been derived. The development of dynamic models for heat exchangers operating in two-phase has focused in three main approaches: lumped-parameter (moving-boundary) [3,5], spatially distributed [6], and system identification techniques [7]. Since the moving-boundary approach is well suited for control-oriented modeling of vapor compression systems [8], in this paper we use this approach to analyze the controllability of a heat exchanger operating under two-phase conditions.

Controllability of linear systems is a well known subject with a large number of references available in the literature [9–11]. Controllability of nonlinear systems has also been studied extensively but there are still some open questions about sufficient conditions for deciding when a nonlinear system is locally or globally controllable [12]. Some of the references from the literature address issues related to controllability of nonlinear time-invariant continuous-time systems of the form $\dot{x} = f(x, u)$ [14,13]. A significant number of references concentrate in controllability of control-affine systems with [15,17–19] or without [16,20] drift. Nonlinear infinite dimensional systems have also been analyzed in [21]. The purpose of this paper is to apply the concepts of nonlinear controllability to the nonlinear model of a heat exchanger operating in two-phase flow.

2. Moving-boundary approach

As mentioned in the previous section, we will analyze the controllability of an evaporator simulated with a lumped-parameter moving-boundary model. This formulation is obtained by dividing the length of the heat exchanger into a two-phase and a single-phase section as shown in Fig. 2. The model does not include condensation on the external surface of the heat exchanger and it follows the derivation found in [3]. The model is based in the work by Wedekind [4] who showed experimentally that the mean void fraction (the volumetric ratio of vapor to liquid) remains relatively invariant in the two-phase region of a heat exchanger under different operating conditions. Some of the assumptions utilized in the derivation of the model include the heat exchanger simulated as a long and thin horizontal tube, the refrigerant flowing through the heat exchanger is modeled as a one-dimensional flow, and the axial heat conduction is negligible.

The following are the governing equations for each section of the heat exchanger:

Node 1 Energy balance

$$Al_1\left(\frac{\mathrm{d}(\rho_l h_l)}{\mathrm{d}t}(1-\bar{\gamma})+\frac{\mathrm{d}(\rho_g h_g)}{\mathrm{d}t}\bar{\gamma}-\frac{\mathrm{d}P}{\mathrm{d}t}\right)+A(1-\bar{\gamma})(\rho_l h_l-\rho_g h_g)\frac{\mathrm{d}l_1}{\mathrm{d}t}=\dot{m}_i h_i-\dot{m}_{int}h_{int}+\alpha_{i1}\pi D_i l_1(T_{w1}-T_{r1}) \quad (1)$$

Mass balance

$$Al_1\frac{\mathrm{d}\rho_l}{\mathrm{d}P}\frac{\mathrm{d}P}{\mathrm{d}t}+A(\rho_l(1-\bar{\gamma})+\rho_g\bar{\gamma}-\rho_g)\frac{\mathrm{d}l_1}{\mathrm{d}t}=\dot{m}_i-\dot{m}_{int} \quad (2)$$

Energy equation at tube wall

$$(C_p\rho A)_w\frac{\mathrm{d}T_{w1}}{\mathrm{d}t}=\alpha_{i1}\pi D_i(T_{r1}-T_{w1})+\alpha_o D_o(T_a-T_{w1}) \quad (3)$$

Node 2 Energy balance

$$Al_2\left(\rho_2\frac{\mathrm{d}h_2}{\mathrm{d}t}-\frac{\mathrm{d}P}{\mathrm{d}t}\right)=\alpha_{i2}\pi D_i l_2(T_{w2}-T_{r2})-\left(\dot{m}_{int}-\rho_g A\frac{\mathrm{d}l_1}{\mathrm{d}t}\right)\frac{h_o-h_{int}}{2}-\dot{m}_o\frac{h_o-h_{int}}{2} \quad (4)$$

Mass balance

$$Al_2\frac{\mathrm{d}\rho_2}{\mathrm{d}t}+A(\rho_g-\rho_2)\frac{\mathrm{d}l_1}{\mathrm{d}t}=\dot{m}_{int}-\dot{m}_o \quad (5)$$

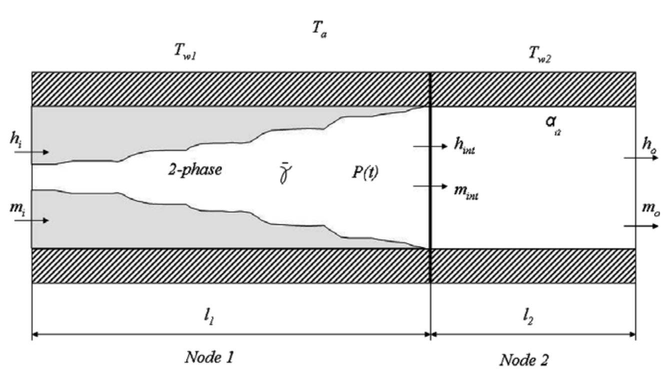

Fig. 2. Schematic of evaporator model.

Energy equation at tube wall

$$(C_p\rho A)_w\left(\frac{\mathrm{d}T_{w2}}{\mathrm{d}t}+\frac{T_{w1}-T_{w2}}{L_2}\frac{\mathrm{d}l_1}{\mathrm{d}t}\right)=\alpha_{i2}\pi D_i(T_{r2}-T_{w2})+\alpha_o\pi D_o(T_a-T_{w2}) \quad (6)$$

These equations can be combined to obtain the nonlinear system.

$$\dot{\mathbf{x}}=\mathbf{Z}(\mathbf{x},\mathbf{u})^{-1}\mathscr{F}(\mathbf{x},\mathbf{u}) \quad (7)$$

where $\mathbf{x}=[\,l_1 \quad P \quad h_o \quad T_{w1} \quad T_{w2}\,]$ are the state variables and $\mathbf{u}=[\,\dot{m}_i \quad h_i \quad \dot{m}_o \quad \alpha_o\,]$ are the manipulated variables and where l_1 is the length of the two-phase region, P is the pressure at the evaporator, h_o is the outlet enthalpy, and T_{w1} and T_{w2} are the wall temperatures at the two-phase and single-phase section, respectively. $\dot{m}_i$ is the inlet mass flow rate, h_i is the inlet enthalpy, $\dot{m}_o$ is the outlet mass flow rate, and α_o is the external heat transfer coefficient. The matrix $\mathbf{Z}$ is non-singular as long as we are operating at conditions for which l_1 and l_2 are different than zero. The coefficients of $\mathbf{Z}(\mathbf{x},\mathbf{u})$ are given in [22].

2.1. Linearized equations

A linearization of this system about an operating point $\mathbf{x^0}$ is performed. The linearized equations take the form

$$\Delta\dot{\mathbf{x}}=\mathbf{A}\Delta\mathbf{x}+\mathbf{B}\Delta\mathbf{u} \quad (8)$$

where

$$\mathbf{A}=[\mathbf{Z}(\mathbf{x},\mathbf{u})|_{x_0,u_0}]^{-1}\left[\frac{\partial\mathscr{F}}{\partial x}\Big|_{x_0,u_0}\right],$$

$$\mathbf{B}=[\mathbf{Z}(\mathbf{x},\mathbf{u})|_{x_0,u_0}]^{-1}\left[\frac{\partial\mathscr{F}}{\partial u}\Big|_{x_0,u_0}\right]$$

It has been shown [9] that using the controllability matrix, $\mathbf{Q_c}$, the linearized system given by Eq. (8) is controllable if and only if rank$(\mathbf{Q_c})=n$ where

$$\mathbf{Q_c}=[\,\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \quad .. \quad \mathbf{A}^{n-1}\mathbf{B}\,] \quad (9)$$

and n is the number of states, in this case is equal to five. By inserting matrices **A** and **B** in Eq. (9), the system described by Eq. (8) is found to be locally controllable. However, for normal operating conditions, as described in section 6, the condition number of the controllability matrix, i.e. the ratio of the largest to smallest singular values, is 8.442×10^9 which shows that large values of the control input are required to control this heat exchanger [2].

3. Extreme cases

It is important to understand how the model presented in Section 2 behaves under different operating conditions, especially at extreme cases. Considering Eqs. (1)–(6) under steady-state operating conditions, we reduce the system of algebraic equations to

$$\dot{m}_{\rm i}(h_{\rm i}-h_{\rm o})=\alpha_{\rm o}D_{\rm o}\pi[(T_{\rm w2}-T_{\rm a})l_2+(T_{\rm w1}-T_{\rm a})l_1] \quad (10)$$

where D_o is the external tube diameter and T_a is the ambient temperature. By noting that $L = l_1 + l_2$ is the total length of the heat exchanger, we can obtain the following expression for the length of the two-phase region (l_1):

$$l_1=\frac{\dot{m}_{\rm i}(h_{\rm i}-h_{\rm o})+\alpha_{\rm o}D_{\rm o}\pi(T_{\rm a}-T_{\rm w2})L}{\alpha_{\rm o}D_{\rm o}\pi(T_{\rm w1}-T_{\rm w2})} \quad (11)$$

For the system to be locally controllable l_1 needs to be greater than zero and it also has to be smaller than the length of the heat exchanger (L).

If we consider the case in which the mass flow rate $\dot{m}_i$ goes to zero, we obtain

$$0<\frac{T_{\rm a}-T_{\rm w2}}{T_{\rm w1}-T_{\rm w2}}<1 \quad (12)$$

For an evaporator operating in steady state, this condition would imply that the wall temperature of the two-phase region, T_{w1}, should be larger than the wall temperature of the single-phase section, T_{w2}, which is not possible. On the other hand, if we consider the condition for which the external heat transfer coefficient (α_o) goes to zero (i.e. insulated tube walls), the difference between the inlet and outlet enthalpies ($h_i - h_o$) would need to go to zero to obtain a finite value. Thus, under this extreme condition, the evaporator is not transferring heat. Therefore the model is not suitable for extreme operating conditions. Utilizing Eq. (11) we obtain that the model works for the ratio between mass flow rate and external heat transfer coefficient given by

$$\Lambda(T_{\rm a}-T_{\rm w1})>\frac{\dot{m}_{\rm i}}{\alpha_{\rm o}}>\Lambda(T_{\rm a}-T_{\rm w2}) \quad (13)$$

where $\Lambda = D_o\pi L/(h_o - h_i)$.

4. Nonlinear controllability

So far, we have shown controllability of the linearized equations around an operating point. We have also analyzed the nonlinear equations under steady-state operating conditions subject to extreme cases of refrigerant mass flow rate and external heat transfer coefficient. Ultimately, we would like to be able to have some notion of the controllability of the original nonlinear equations without having to restrict ourselves to linearized models or particular cases of the nonlinear model. In this section of the paper we apply concepts of Lie algebras to analyze the local controllability of the nonlinear equations.

4.1. Lie algebras

Many physical systems can be simulated and analyzed with models of the form,

$$\Sigma:\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x})+\mathbf{g}_1(\mathbf{x})u_1+\cdots+\mathbf{g}_m(\mathbf{x})u_m \quad (14)$$

where M is a smooth manifold, and $\mathbf{x} \in M$; $\mathbf{f}, \mathbf{g}_1, \ldots, \mathbf{g}_m$ are smooth vector fields on M. The control variable u_i represents the external inputs and $\mathbf{x}$ represent the state variables. When $\mathbf{f} = 0$ these systems are called control-affine without drift because when u is zero the state does not drift, but, instead remains constant [14]. When $\mathbf{f} \neq 0$ then the system is called control-affine with drift. The set of all (smooth) vector fields on a given $M \subseteq \mathbb{R}^n$ is denoted by $V(M)$.

Definition 4.1. The *Lie bracket* of $\mathbf{f}$, $\mathbf{g} \in V(M)$ is $[\mathbf{f},\mathbf{g}] = \frac{\partial \mathbf{g}}{\partial \mathbf{x}}\mathbf{f} - \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{g}$.

Definition 4.2. A *Lie algebra* (of vectors fields on M) is a linear subspace $S \subseteq V(M)$ that is closed under the Lie bracket operation, that is, $[\mathbf{f},\mathbf{g}] \in S$ whenever $\mathbf{f}$ and $\mathbf{g}$ are in S.

Let $\mathscr{C}$ denote the smallest subalgebra of $V(M)$ that contains $\mathbf{f}, \mathbf{g}_1, \ldots, \mathbf{g}_m$. If $\dim(\mathscr{C}) = \dim(M)$ at a point $\mathbf{x^0}$, then the system described by Eq. (14) satisfies the *Lie Algebra Rank Condition* ("LARC") at $\mathbf{x^0}$ [17].

Definition 4.3. A system is small-time locally controllable ("STLC", or simply "controllable") if the set of states that are reachable in time T contains a neighborhood of $\mathbf{x^0}$ for all $T > 0$.

Theorem 4.4. *If the system described by Eq. (14) is such that* $\mathbf{f} \equiv 0$ *and satisfies the LARC at a point* $\mathbf{x^0}$ *then it is STLC from* $\mathbf{x^0}$.

For systems for which $\mathbf{f} \neq 0$ the analysis of controllability requires a stronger result [17,23]. We define the concepts of "good" and "bad" brackets [15]. For a bracket term B, we define $\delta_i(B)$ as the number of times the indeterminate X_i appears in B, and the degree of B is $\sum_{i=0}^{n}\delta_i(B)$. B is called a "bad" bracket if $\delta_0(B)$ is odd and $\delta_i(B)$ is even for all $i \in 1, \ldots, n$, and B is a "good" bracket otherwise. A "bad" bracket B is "neutralized" at a state $\mathbf{x^0}$ if B, evaluated at $\mathbf{x^0}$, is the linear combination of "good" brackets of lower degree evaluated at $\mathbf{x^0}$.

Therefore, and as stated in [17]

$\mathbf{f}$	is bad
$[\mathbf{g}_1,\mathbf{g}_2]$	is good
$[\mathbf{f},[\mathbf{g}_1,\mathbf{g}_2]]$	is bad
$[\mathbf{g}_1,[\mathbf{g}_1,\mathbf{g}_2]]$	is good

Theorem 4.5. *If a system* Σ *satisfies LARC and all bad brackets are spanned by lower degree good brackets, then* Σ *is STLC.*

4.2. Analysis of nonlinear model

The nonlinear model described by Eq. (7) can be rearranged and written in terms of vector fields to form a control-affine model with drift of the form $\dot{x} = \mathbf{f} + \mathbf{g}_1u_1 + \mathbf{g}_2u_2 + \mathbf{g}_3u_3 + \mathbf{g}_4u_4$

$$\begin{pmatrix} \dot{l}_1 \\ \dot{P} \\ \dot{h}_o \\ \dot{T}_{w1} \\ \dot{T}_{w2} \end{pmatrix} = \begin{pmatrix} z_{11}\kappa_1 - z_{12}\kappa_2 \\ z_{21}\kappa_1 - z_{22}\kappa_2 \\ z_{31}\kappa_1 - z_{32}\kappa_2 \\ z_{44}\alpha_{i1}\pi D_i(T_{r1} - T_{w1}) z_{51}\kappa_1 - z_{52}\kappa_2 \end{pmatrix} + \begin{pmatrix} z_{13} - z_{11}h_g \\ z_{23} - z_{21}h_g \\ z_{33} - z_{31}h_g \\ 0 \\ z_{53} - z_{51}h_g \end{pmatrix}\dot{m}_i + \begin{pmatrix} z_{11}h_g \\ z_{21}h_g \\ z_{31}h_g \\ 0 \\ z_{51}h_g \end{pmatrix}\phi_i + \begin{pmatrix} z_{12}(h_g - h_o) - z_{13} \\ z_{22}(h_g - h_o) - z_{23} \\ z_{32}(h_g - h_o) - z_{33} \\ 0 \\ z_{52}(h_g - h_o) - z_{53} \end{pmatrix}\dot{m}_o + \begin{pmatrix} 0 \\ 0 \\ 0 \\ z_{44}\pi D_o(T_a - T_{w1}) \\ z_{55}\pi D_o(T_a - T_{w2}) \end{pmatrix}\alpha_o \quad (15)$$

where $\kappa_1 = \alpha_{i1}\pi D_i l_1(T_{w1} - T_{r1})$, $\kappa_2 = \alpha_{i2}\pi D_i(L - l_1)(T_{w2} - T_{r2})$, $\phi_i = m_i h_i$, and the coefficients z_{ij} are obtained from matrix $\mathbf{Z}^{-1}$ in Eq. (7). Since a model must be linear with respect to the manipulated variables to be classified as control-affine, we have chosen ϕ_i instead of h_i as one of the manipulated variables. The manipulated variables $\dot{m}_i$ and ϕ_i are still independent from each other since h_i provides an additional degree of freedom.

To test for the controllability rank condition we use $(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3, \mathbf{g}_4, \mathbf{g}_5)$.

In this case $\mathbf{g}_5 = [\mathbf{g}_3, \mathbf{g}_4] = \left(0, 0, 0, 0, g_{54}\frac{g_{33}}{T_{w2}}\right)'$ where $'$ denotes the transpose and g_{33} and g_{54} are obtained from the matrix of vector fields that is formed from the last four column vectors in Eq. (15) plus $\mathbf{g}_5$

$$\mathbf{W}(\mathbf{x}^0) = (\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3, \mathbf{g}_4, \mathbf{g}_5) = \begin{pmatrix} g_{11} & g_{12} & g_{13} & 0 & 0 \\ g_{21} & g_{22} & g_{23} & 0 & 0 \\ g_{31} & g_{32} & g_{33} & 0 & 0 \\ 0 & 0 & 0 & g_{44} & 0 \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{55} \end{pmatrix} \quad (16)$$

The determinant of matrix $\mathbf{W}(\mathbf{x}^0)$ is

$$\det(\mathbf{W}(\mathbf{x}^0)) = [g_{22}(g_{11}g_{33} - g_{31}g_{13}) + g_{32}(g_{21}g_{13} - g_{11}g_{23}) + g_{12}(g_{31}g_{23} - g_{21}g_{33})]g_{44}g_{55} \quad (17)$$

This determinant is in general different than zero, so the controllability matrix has a generic rank of five and the system satisfies the LARC condition. However, there may exist points $\mathbf{x}^0$ for which $\mathbf{W}(\mathbf{x}^0)$ is singular. This possibility must be checked for particular $\mathbf{x}^0$ [16]. We consider now some of the brackets.

$$[\mathbf{g}_1, \mathbf{g}_2] = (*, *, *, 0, *)'; \quad [\mathbf{g}_3, \mathbf{g}_4] = (0, 0, 0, 0, *)'$$
$$[\mathbf{f}, \mathbf{g}_1] = (*, *, *, 0, *)'; \quad [\mathbf{f}, \mathbf{g}_4] = (*, *, *, *, *)'$$
$$[\mathbf{g}_1, [\mathbf{f}, \mathbf{g}_1]] = (*, *, *, 0, *)'; \quad [\mathbf{g}_4, [\mathbf{f}, \mathbf{g}_4]] = (0, 0, 0, 0, *)'$$

where $*$ denotes a number different than zero. The "bad" brackets are spanned by lower order "good" brackets. Thus, by means of Theorem 4.5, we have that the system described by Eq. (7) is STLC at $\mathbf{x}^0$, unless $\mathbf{W}(\mathbf{x}^0)$ becomes singular at that operating point.

5. LQ optimal control

The control of the evaporator can be done using different techniques. Steering control can be applied directly to the control-affine with drift model [26,27]. However, this type of control is not commonly used in industry for residential or mobile air conditioning systems. In this section we apply a more standard approach and we develop a controller for the linearized equations.

Consider a linear system of the form.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (18)$$

We define the augmented system as

$$\widehat{A} = \begin{pmatrix} A & 0 \\ -C & 0 \end{pmatrix}, \quad \widehat{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \quad \widehat{G} = \begin{pmatrix} 0 \\ I \end{pmatrix} \quad (19)$$

to form the system

$$\begin{pmatrix} \dot{x} \\ \dot{w} \end{pmatrix} = \widehat{A}\begin{pmatrix} x \\ w \end{pmatrix} + \widehat{B}u + \widehat{G}r \quad (20)$$

where r is the reference command and $\dot{w} = r - Cx$. We define the error

$$e = \begin{pmatrix} r - Cx \\ w \end{pmatrix} = Mr + Hx \quad (21)$$

where

$$M = \begin{pmatrix} I \\ 0 \end{pmatrix}, \quad H = \begin{pmatrix} -C & 0 \\ 0 & I \end{pmatrix} \quad (22)$$

We find the linear optimal tracker that minimizes the function

$$J = \frac{1}{2}\int_0^\infty (e'Qe + u'Ru)\,\mathrm{d}t \quad (23)$$

where $Q \geqslant 0$ and $R > 0$. Following the procedure stated in [24,25] we obtain

$$K = R^{-1}\widehat{B}'S \quad (24)$$

where S is the solution to the Riccati equation

$$S\widehat{A} + \widehat{A}'S + S\widehat{B}R^{-1}\widehat{B}'S + Q = 0 \quad (25)$$

The control law becomes $u = -K\hat{x}$ and the closed loop system is as follows:

$$\dot{\hat{x}} = \begin{pmatrix} A - BK_x & -BK_w \\ -C & 0 \end{pmatrix}\hat{x} + \begin{pmatrix} 0 \\ I \end{pmatrix}r \quad (26)$$

where $K = [K_x \;\; K_w]$.

6. Numerical simulations

6.1. Open loop control

We now compare the behavior of the nonlinear and linearized models subject to open loop control. Carbon dioxide is used as refrigerant and we linearize the equations with respect to an arbitrary fixed operating point for which the length of the two-phase section is $l_1 = 75\%$ and it is based on the total length of the coil, the evaporating pressure is $P = 3455$ kPa, the enthalpy at the outlet of the coil is $h_o = 447$ kJ/kg, the wall temperature at the two-phase region is $T_{w1} = 0.07$ °C, and at the single-phase region is $T_{w2} = 9.58$ °C. There are four manipulated variables that can be varied, i.e. the inlet mass flow rate $\dot{m}_i$, the inlet enthalpy h_i, the outlet mass flow rate $\dot{m}_o$, and the external heat transfer coefficient α_o. Many air conditioning systems allow a manual selection of the air flow rate across the evaporator. We will consider this case for our analysis. An arbitrary 50% change in the external heat transfer coefficient is applied to the nonlinear and linearized equations at $t = 200$ s. The values of the other three manipulated variables are kept constant. Fig. 3 shows the comparison between the responses of the two models. The linearized system simulates the dynamics of the nonlinear system accurately. The main differences are due to the evaluation of the fluid properties at different temperatures in the nonlinear model. For the linearized model, matrices A and B in Eq. (8) are evaluated at a fixed operating point. It is noticed that at these operating conditions, the singular values of matrix A show multiple time scales as reported by other researchers [3,8].

6.2. LQ control: Step change in P using linear model

The open loop control analysis verified that the linear model represents the dynamics of the nonlinear model accurately. We now use the derivations from the previous section to obtain a LQ controller for the linear model. Due to the large difference in magnitude of the variables involved, the values of matrices R and Q need to be scaled. Similar operating conditions as in the open loop control case are used but in this case we vary the reference evaporating pressure from $P_1 = 3520$ kPa to $P_2 = 3700$ kPa. Fig. 4 shows the behavior of the controlled variables. In spite of the strong coupling between the state variables, the controller is able to keep the controlled variables at the desired values. Due to the increase in the evaporator pressure, the temperature of the refrigerant is increased from 0.376 °C to 2.27 °C. Since there is only a small difference between the reference temperature for the wall in the two-phase region, T_{w1}, and the saturated temperature of the refrigerant at the operating pressure, the controller requires a long time to drive the wall temperature of the two-phase region back to the reference value. As it is expected for an evaporator, the reference of the wall temperature has to remain at a value larger than the saturation temperature of the refrigerant at the evaporating pressure.

6.3. LQ control: Step change in h_o using linear model

A step change in the reference for the evaporating pressure modifies the saturating temperature and therefore affects the wall temperature and the length of the two-phase section. Other reference parameters have a smaller impact

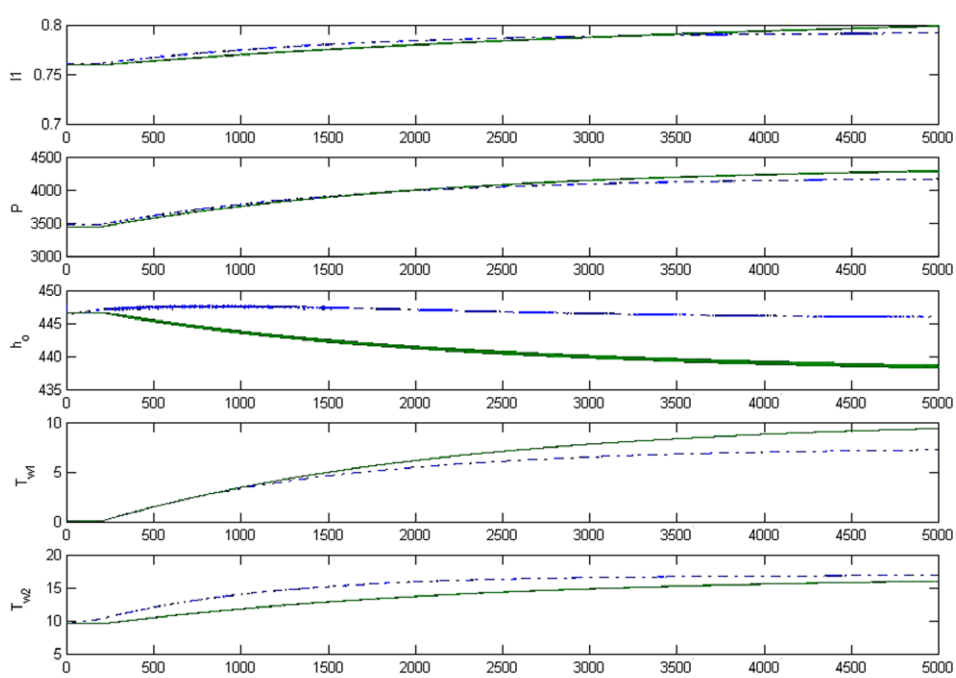

Fig. 3. Open loop response. Solid line – linearized equations. Dashed line – nonlinear equations.

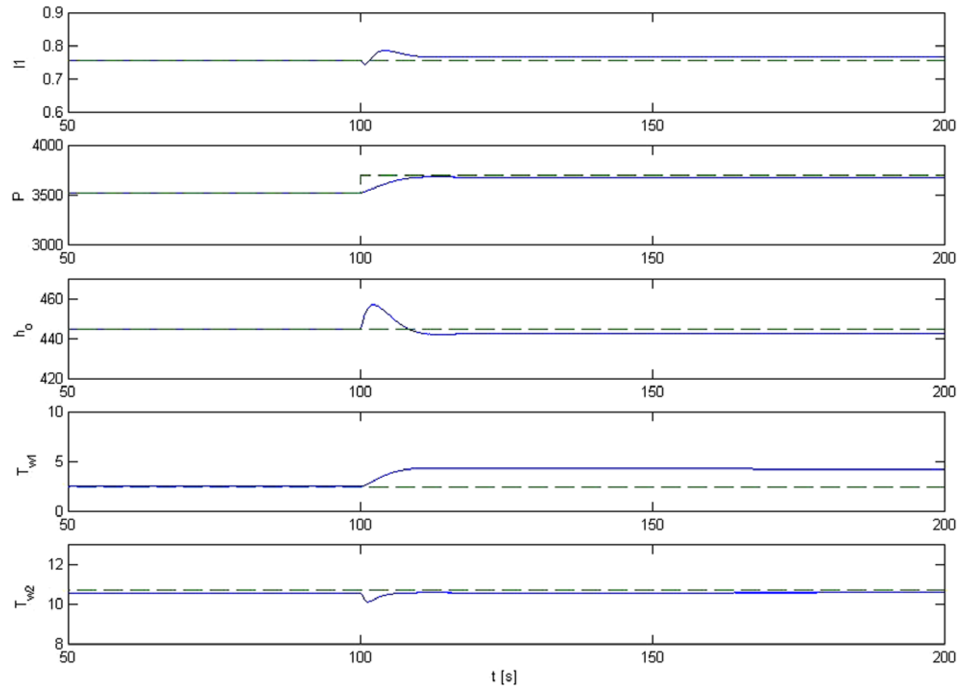

Fig. 4. Step change in evaporator operating pressure. Solid line – controlled variables, Dashed line – reference.

on other variables when modified. For instance, a step change in the reference for the outlet enthalpy has a small impact on the other controlled variables. Fig. 5 shows the behavior of the linearized model subject to LQ control for a change in the reference for the outlet enthalpy. The reference is tracked accurately and no significant disturbance is observed in the other controlled variables.

6.4. LQ control: Step change in l_1 using nonlinear model

In the previous sections it is shown that the LQ controller works well with the linearized model. We now test the LQ controller with the nonlinear model by performing a step change in the length of the two-phase region. We start from the same initial conditions used to generate Figs. 4

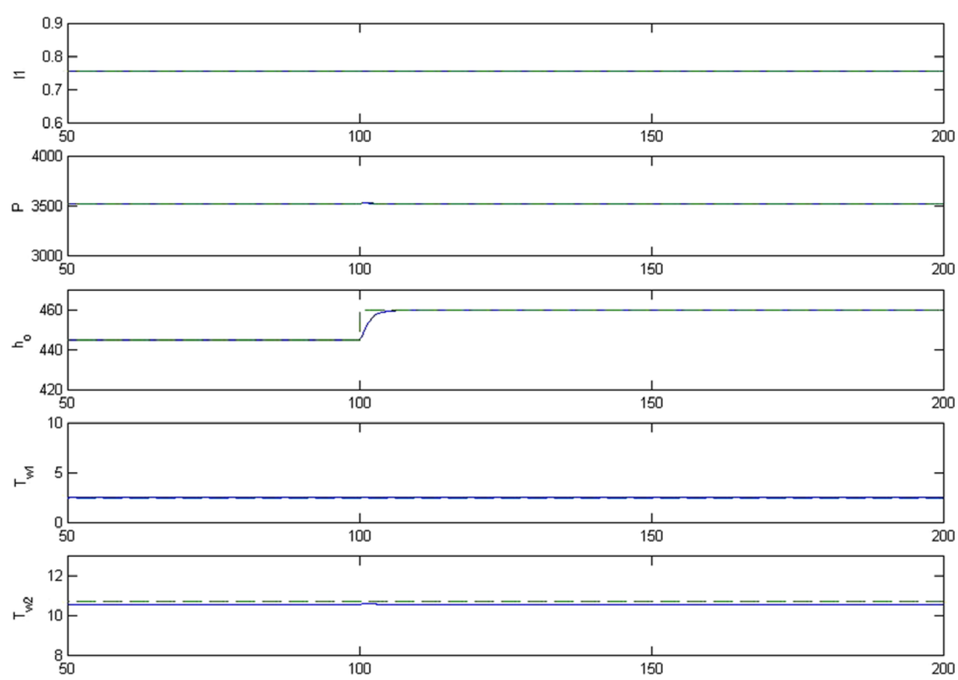

Fig. 5. Step change in outlet enthalpy. Solid line – controlled variables, Dashed line – reference.

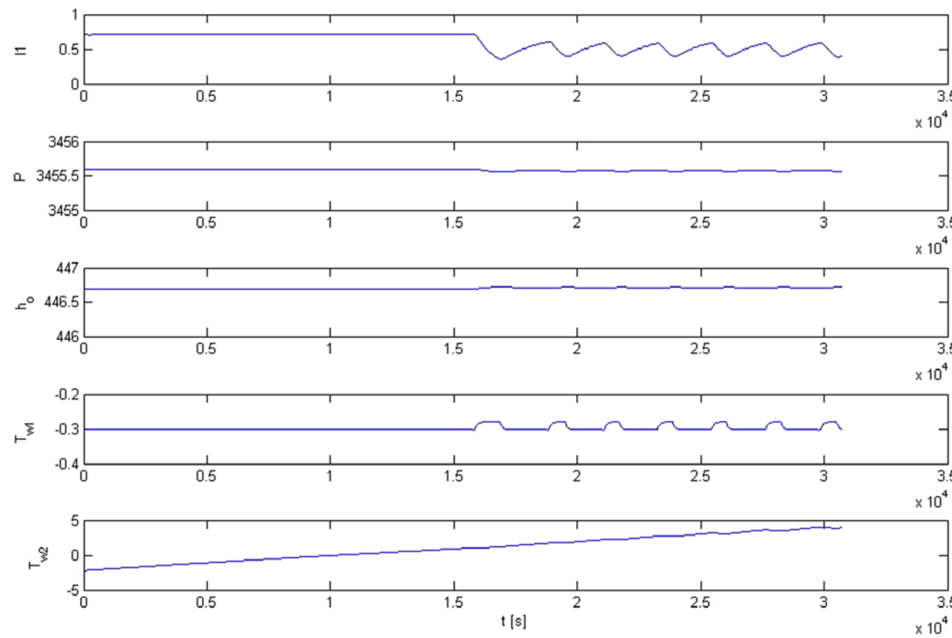

Fig. 6. Step change in outlet length of two-phase section using nonlinear model.

and 5, so that l_1 starts with a value of 75% of the total length of the heat exchanger. At time equal to 11,000 [s] we change the reference to 50% keeping the other four reference values constant. The results are shown in Fig. 6. The controller drives the system to the new reference value but oscillations occur in the length of the two-phase region. This shows the differences between the linearized and nonlinear models. Also, due to the large difference in the magnitude of the variables involved (i.e. l_1, P, h_o, T_{w1}, and T_{w2}) and the arbitrary values chosen for matrices Q and R in Eq. (23), the controller has better performance tracking the reference values of P and h_o compared to l_1, T_{w1}, and T_{w2}. This suggests that the model should be normalized to avoid the large difference in magnitude of the variables involved.

7. Conclusions

The analysis of the controllability of a heat exchanger operating under two-phase conditions is performed. The nonlinear model is linearized and local controllability is shown around a fixed operating point. The behavior of the dynamic model is also analyzed under extreme conditions of mass flow rate and external heat transfer coefficient. The nonlinear model is written in the form of a control-affine system with drift. The system is shown to be STLC. Finally, numerical simulations are performed that show that the linear system describes the dynamic behavior of the nonlinear model accurately. A LQ controller is developed for the linearized equations and step changes in the reference evaporating pressure and outlet enthalpy are applied. The results show that changing the reference value of the evaporating pressure has a larger effect on the other states than a change in the reference value of the outlet enthalpy. Finally, the LQ controller is used with the nonlinear model to generate a step change in the length of the two-phase flow region of the heat exchanger.

Acknowledgements

The author would like to thank Professor Mihir Sen for providing several comments about the manuscript.

References

[1] B.P. Rasmussen, A.G. Alleyne, A.B. Musser, Model-driven system identification of transcritical vapor compression systems, IEEE Trans. Control Syst. Technol. 13 (3) (2005) 444–451.
[2] S. Alotaibi, M. Sen, B. Goodwine, K.T. Yang, Controllability of cross-flow heat exchangers, Int. J. Heat Mass Transfer 47 (2004) 913–924.
[3] X-D. He, S. Liu, H.H. Asada, Modeling of vapor compression cycles for multivariable feedback control of HVAC systems, ASME J. Dynam. Syst. Measur. Control 119 (2) (1997) 183–191.
[4] G.L. Wedekind, B.L. Bhatt, B.T. Beck, A system mean void fraction model for predicting various transient phenomena associated with two-phase evaporating and condensing flows, Int. J. Multiphase Flow 4 (1978) 97–114.
[5] E.W. Grald, J.W. MacArthur, A moving-boundary formulation for modeling time-dependent two-phase flows, Int. J. Heat Fluid Flow 13 (3) (1992) 266–272.
[6] J.W. MacArthur, E.W. Grald, Unsteady compressible two-phase flow model for predicting cyclic heat pump performance and a comparison with experimental data, Int. J. Refrig. 12 (1989) 29–41.
[7] B.P. Shah, B.P. Rasmussen, A.G. Alleyne, Application of a multivariable adaptive control strategy to automotive air conditioning systems, Int. J. Adaptive Control Signal Process. 18 (2004) 199–221.
[8] B.P. Rasmussen, A.G. Alleyne, Control-oriented modeling of transcritical vapor compression systems, ASME J. Dynam. Syst. Control 13 (2004) 54–64.
[9] B.M. Chen, Z. Lin, J. Shamash, Linear Systems Theory: A Structural Decomposition Approach, Birkhauser, Boston, 2004.

[10] P.J. Antsaklis, A.N. Michel, Linear Systems, McGraw-Hill, 1997.
[11] D.L. Russell, Controllability and stabilizability theory for linear partial differential equations: recent progress and open questions, SIAM Rev. 20 (4) (1978) 639–739.
[12] E. Sontag, Controllability is harder to decide than accessibility, SIAM J. Control Opt. 26 (1988) 1106–1118.
[13] R. Hermann, A.J. Krener, Nonlinear controllability and observability, IEEE Trans. Automat. Control AC-22 (1977) 728–740.
[14] E.D. Sontag, Mathematical Control Theory: Deterministic Finite Dimensional Systems, Springer, New York, 1998.
[15] H.J. Sussmann, A general theorem on local controllability, SIAM J. Control Opt. 25 (1) (1987) 158–194.
[16] M.A. Henson, D.E. Seborg, Nonlinear Process Control, Prentice-Hall PTR, 1997.
[17] M.B. McMickell, B. Goodwine, Reduction and nonlinear controllability of symmetric distributed robotic systems with drift, in: Proceedings of 2002 IEEE International Conference on Robotics & Automation, Washington DC, 2002, pp. 3454–3460.
[18] A.M. Bloch, M. Reyhanoglu, N.H. McClamroch, Control and stabilization of nonholonomic dynamic systems, IEEE Trans. Automat. Control 37 (11) (1992) 1746–1757.
[19] K. Lynch, M. Mason, Dynamic nonprehensile manipulation: controllability, planning and experiments, Int. J. Robot. Res. 18 (1) (1999) 64–92.
[20] S. Sastry, Nonlinear Systems: Analysis, Stability and Control, Springer-Verlag, 1999.
[21] A. Doubova, E. Fernandez-Cara, M. Gonzalez-Burgos, E. Zuazua, On the controllability of parabolic systems with a nonlinear term involving the state and the gradient, SIAM J. Control Opt. 21 (5) (1983) 798–819.
[22] Rasmussen, B.P., Control-oriented modeling of transcritical vapor compression systems, M.S. Thesis, University of Illinois, Urbana, IL, 2002.
[23] H.J. Sussmann, Lie brackets and local controllability: a sufficient condition for scalar-input systems, SIAM J. Control Opt. 41 (3) (2002) 686–712.
[24] A.E. Bryson, Dynamic Optimization, Addison Wesely Longman, Inc., 1999.
[25] K.P Groves, D.O. Sigthorsson, A. Serrani, S. Yurkovich, M.A. Bolender, D.B. Doman, Reference command tracking for a linearized model of an air-breathing hypersonic vehicle, in: Proceedings of AIAA Guidance, Navigation, and Control Conference and Exhibit, San Francisco, California, Paper No. AIAA 2005-6144, 2005.
[26] Fazal-ur-Rehman, Discontinuous steering control for nonholonomic systems with drift, Nonlinear Analysis 63 (2005) 311–325.
[27] Fazal-ur-Rehman, Steering control of nonholonomic systems with drift: the extended nonholonomic double integrator example, Nonlinear Anal. 62 (2005) 1498–1515.